Energy conditions and classical scalar fields

S. Bellucci[1] and V. Faraoni[1,2]

[1] *INFN-Laboratori Nazionali di Frascati, P.O. Box 13, I-00044 Frascati, Roma, Italy*

[2] *Physics Department, University of Northern British Columbia*
3333 University Way, Prince George, B.C., V2N 4Z9, Canada

Abstract

Attention has been recently called upon the fact that the weak and null energy conditions and the second law of thermodynamics are violated in wormhole solutions of Einstein's theory with classical, nonminimally coupled, scalar fields as material source. It is shown that the discussion is only meaningful when ambiguities in the definitions of stress-energy tensor and energy density of a nonminimally coupled scalar are resolved. The three possible approaches are discussed with emphasis on the positivity of the respective energy densities and covariant conservation laws. The root of the ambiguities is traced to the energy localization problem for the gravitational field.

1 Introduction

Scalar fields are ubiquitous in particle physics and in cosmology: examples are the Higgs boson of the Standard Model, the string dilaton, the superpartner of spin 1/2 particles in supergravity, and the Brans-Dicke field [1]; or the inflaton field and the quintessence scalar field in cosmology [2, 3].

It is natural that a scalar field ϕ living in a curved space couple explicitly to the Ricci curvature R of spacetime, as described by the action[1]

$$S = \int d^4x\sqrt{-g}\left[\left(\frac{1}{2\kappa} - \frac{\xi\phi^2}{2}\right)R - \frac{1}{2}g^{ab}\nabla_a\phi\nabla_b\phi - V(\phi)\right] + S_m \; , \tag{1.1}$$

where $\kappa \equiv 8\pi G$ (G being Newton's constant), $V(\phi)$ is the scalar field potential, the dimensionless coupling constant ξ describes the explicit nonminimal coupling (hereafter referred to as NMC), and S_m is the action for all forms of matter different from ϕ.

NMC is introduced by first loop corrections: even if a classical scalar field theory is minimally coupled (i.e. $\xi = 0$), first loop corrections shift the value of the coupling ξ to a nonzero one [5, 6], typically of order 10^{-1}-10^{-2} [7]. A nonzero ξ is also required by independent arguments in specific high energy theories (see [8, 9] for reviews). Even at the classical level, and in a minimalist's point of view restricted to general relativity, the Einstein equivalence principle [10] forces the coupling to be nonminimal.[2] More precisely, the consideration of a conformal coupling (i.e. $\xi = 1/6$) is required in this case [13, 14]. [3]

In short, NMC is an unavoidable feature of scalar field physics in curved spaces. Even without knowledge of this fact, it would be wise to incorporate NMC in a scalar field theory due to the fact that NMC can have a dramatic effect on the solutions of the theory and the physics that they describe. As an example, consider the well known chaotic inflationary scenario with the potential $V = \lambda\phi^4$; the inclusion of NMC with a value of ξ as small as 10^{-3} prevents the existence of inflationary solutions unless an unacceptable fine-tuning is allowed, thus ruining an inflationary scenario which is very successful for $\xi = 0$ [16]. NMC is relevant in cosmology (during inflation and the quintessence era) [17], quantum cosmology [18], and in the physics of boson stars [19] and wormholes [20]. Recently[4], it was pointed out that the innocent-looking theory described by the action (1.1) leads to negative energy fluxes [22], opening the door to exotica such as traversable wormholes [22], warp drives [23], and time

[1] We adopt the notations and conventions of Ref. [4]; in particular, conformal coupling corresponds to $\xi = +1/6$ in these notations.

[2] The principle was advocated in a proposal of aconsistent quantum gravity [11]. For the consequences of the tests of the principle in a specific framework, see e.g. [12].

[3] In a different context, superconformal invariance was proposed to explain why newtonian gravity shadows cosmological constant effects [15].

[4] See Ref. [21] for an earlier report of this phenomenon.

machines [20]. Even worse, the second law of thermodynamics is put in jeopardy by the theory (1.1) [24]. Note that the system described by eq. (1.1) does not have quantum nature, such as those often considered in the literature and for which violations of the weak and null energy conditions are temporary but do not persist on average; the theory (1.1) is, instead, a classical one.

The negative energy problem is actually much more general than the theory (1.1): it appears in Brans-Dicke and scalar-tensor theories, in higher derivatives theories of gravity, in the low energy limit of string theories and also in brane world theory[5]. In this paper, however, we mainly deal with the NMC case.

It turns out that, with NMC, there are ambiguities in the definition of the scalar field stress-energy tensor and, as a consequence, in the definition of energy density, pressure, and equation of state. In order to be able to speak of energy conditions, a specific choice of definitions of energy density and pressure must be made; this need is made clear in Sec. 2 of this paper. In addition, conservation laws for all the possible versions of stress-energy tensors (there are three of them) are discussed.

In Sec. 3, the violation of weak and null energy conditions is discussed, and it is pointed out that these violations are not universal, i.e. there are physically interesting situations in which the energy density (defined in a meaningful way) cannot be negative even in the presence of NMC. There are also situations in which negative energies are possible but do not lead to runaway solutions and dynamical catastrophes. And there are situations, which have been pointed out in the literature, in which negative energies become embarassing for a theory with a generalized coupling between the scalar and the gravitational fields [27].[6] A possible cure, albeit a drastic one, for the problem of negative energies with NMC is pointed out in Ref. [24] (see also the references in [27]), and consists in the reformulation of the theory in the Einstein conformal frame, on the lines of what is done for scalar-tensor theories [27]. This possibility is not surprising since the action (1.1) can be rewritten as a scalar-tensor theory by means of a suitable scalar field redefinition. Whether this way out of the problem is needed, or at all satisfactory, will be left to the judgment of the reader.

2 Ambiguities in the stress-energy tensor

In this section we discuss the three possible and inequivalent ways of writing the field equations, the corresponding ambiguity in the definition of the energy-momentum tensor of the scalar field (and therefore of the energy density, pressure, and equation of state), and the corresponding covariant conservation properties in the presence of NMC.

[5]This is not surprising, since the Randall-Sundrum model somehow reduces to a Brans-Dicke theory [25, 26], which suffers from negative kinetic energies.

[6]For a low-energy theorem in gravity coupled to scalar matter, see. e.g. [28].

The field equations derived by varying the action (1.1) are

$$G_{ab} = \kappa \left[\nabla_a\phi\nabla_b\phi - \frac{1}{2}g_{ab}\nabla^c\phi\nabla_c\phi - V(\phi)\, g_{ab} + \xi\left(g_{ab}\Box - \nabla_a\nabla_b\right)(\phi^2) + \xi\phi^2 G_{ab} + T_{ab}^{(m)} \right] , \tag{2.1}$$

where

$$T_{ab}^{(m)} = -\frac{2}{\sqrt{-g}}\frac{\delta S_m}{\delta g^{ab}} \tag{2.2}$$

is the usual stress-energy tensor of ordinary matter (different from ϕ), which does not explicitly depend on ϕ. There are now three ways to proceed, corresponding to different ways of rewriting the field equations (2.1).

2.1 Procedure à la Callan-Coleman-Jackiw

This procedure follows the one originally adopted by Callan, Coleman and Jackiw [29] for conformal coupling. NMC is best known after this work, although it had been introduced earlier in a different context [30]. One writes the Einstein equations (2.1) as

$$G_{ab} = \kappa\left(T_{ab}^{(I)} + T_{ab}^{(m)} \right) , \tag{2.3}$$

where

$$T_{ab}^{(I)} = \nabla_a\phi\nabla_b\phi - \frac{1}{2}g_{ab}\nabla^c\phi\nabla_c\phi - V(\phi)\, g_{ab} + \xi\left(g_{ab}\Box - \nabla_a\nabla_b\right)(\phi^2) + \xi\phi^2 G_{ab} \tag{2.4}$$

contains the "geometric" contribution $\xi\phi^2 G_{ab}$ incorporating the Einstein tensor G_{ab}.

Let us discuss the conservation properties of $T_{ab}^{(I)}$: the contracted Bianchi identities $\nabla^b G_{ab} = 0$ yield

$$\nabla^b T_{ab}^{(I)} + \nabla^b T_{ab}^{(m)} = 0 . \tag{2.5}$$

Since $T_{ab}^{(m)}$ does not depend on ϕ and $\nabla^b T_{ab}^{(m)}$ vanishes when the field ϕ is switched off, one concludes that $T_{ab}^{(m)}$ and $T_{ab}^{(I)}$ are covariantly conserved separately,

$$\nabla^b T_{ab}^{(m)} = 0 , \tag{2.6}$$

$$\nabla^b T_{ab}^{(I)} = 0 . \tag{2.7}$$

The energy density measured by an observer with four-velocity u^c ($u_c u^c = -1$) is, in this approach,

$$\rho_{(total)}^{(I)} \equiv \left[T_{ab}^{(I)} + T_{ab}^{(m)} \right] u^a u^b = \rho_\phi^{(I)} + \rho^{(m)} , \tag{2.8}$$

where

$$\rho_\phi^{(I)} = T_{ab}^{(I)} u^a u^b = (u^c \nabla_c \phi)^2 + \frac{1}{2} \nabla^c \phi \nabla_c \phi + V(\phi) - \xi \Box (\phi^2) - \xi u^a u^b \nabla_a \nabla_b (\phi^2) + \xi \phi^2 G_{ab} u^a u^b \ , \quad (2.9)$$

and $\rho^{(m)} = T_{ab}^{(m)} u^a u^b$ as usual. The trace of $T_{ab}^{(I)}$ reduces to

$$T^{(I)} = -\partial^c \phi \, \partial_c \phi - 4V(\phi) + 3\xi \Box (\phi^2) - \xi R \phi^2 \ . \quad (2.10)$$

2.2 The effective coupling approach

The second way of proceeding is to rewrite the field equations (2.1) by taking the term $\kappa \xi \phi^2 G_{ab}$ to their left hand side,

$$\left(1 - \kappa \xi \phi^2\right) G_{ab} = \kappa \left(T_{ab}^{(II)} + T_{ab}^{(m)}\right) \ , \quad (2.11)$$

where

$$T_{ab}^{(II)} = \nabla_a \phi \nabla_b \phi - \frac{1}{2} g_{ab} \nabla^c \phi \nabla_c \phi - V(\phi) \, g_{ab} + \xi \left(g_{ab} \Box - \nabla_a \nabla_b\right) (\phi^2) = T_{ab}^{(I)} - \xi \phi^2 G_{ab} \ , \quad (2.12)$$

and to further divide eq. (2.11) by the factor $1 - \kappa \xi \phi^2$, obtaining

$$G_{ab} = \kappa_{eff} \left(T_{ab}^{(II)} + T_{ab}^{(m)}\right) \ , \quad (2.13)$$

where

$$\kappa_{eff}(\phi) \equiv \frac{\kappa}{1 - \kappa \xi \phi^2} \quad (2.14)$$

is an effective gravitational coupling for *both* $T_{ab}^{II)}$ and $T_{ab}^{(m)}$. This procedure is analogous to the familiar identification of the Brans-Dicke scalar field ϕ_{BD} with the inverse of an effective gravitational constant ($G_\phi = \phi_{BD}^{-1}$) in the gravitational sector of the Brans-Dicke action

$$S_{BD} = \int d^4 x \sqrt{-g} \left(\phi_{BD} R - \frac{\omega}{\phi_{BD}} \nabla^a \phi_{BD} \, \nabla_a \phi_{BD}\right) \ . \quad (2.15)$$

It is clear that the division by the factor $1 - \kappa \xi \phi^2$ leads to loss of generality for $\xi > 0$; solutions of eq. (2.1) with scalar field attaining the critical values

$$\pm \phi_c \equiv \pm \frac{1}{\sqrt{\kappa \xi}} \qquad (\xi > 0) \ , \quad (2.16)$$

are missed when considering eq. (2.13) in the effective coupling approach. For example, solutions in which ϕ is constant and equal to the critical values (2.16) appear in models of wormholes [22] and as special heteroclinics in the phase space of scalar field cosmology [31] (see the Appendix for a discussion of solutions with constant scalar field). In the effective coupling approach these solutions, which are important ones for wormholes and cosmology, are missed and can only be recovered by going back to the primitive form (2.1) of the field equations. In other words, eq. (2.13) is less general than eq. (2.1).

Let us discuss now the conservation laws for $T_{ab}^{(II)}$; the contracted Bianchi identities imply that

$$\nabla^b \left(T_{ab}^{(II)} + T_{ab}^{(m)} \right) = \frac{-2\kappa\xi\phi}{1-\kappa\xi\phi^2} \left(\nabla^b \phi \right) \left(T_{ab}^{(II)} + T_{ab}^{(m)} \right) . \tag{2.17}$$

By reasoning as in the previous case, one obtains $\nabla^b T_{ab}^{(m)} = 0$ and

$$\nabla^b T_{ab}^{(II)} = \frac{-2\kappa\xi\phi}{1-\kappa\xi\phi^2} \left(\nabla^b \phi \right) \left(T_{ab}^{(II)} + T_{ab}^{(m)} \right) . \tag{2.18}$$

One can have a better idea of the consequences of the conservation law (2.18) by considering ordinary matter in the form of a dust fluid with corresponding energy-momentum tensor $T_{ab}^{(m)} = \rho v_a v_b$, where v^c is the dust four-velocity. Eq. (2.18) then yields

$$\left(\frac{d\rho^{(m)}}{d\lambda} + \rho^{(m)} \nabla^b v_b + \frac{2\kappa\xi\rho^{(m)}\phi}{1-\kappa\xi\phi^2} \frac{d\phi}{d\lambda} \right) v_a + \rho^{(m)} \frac{Dv_a}{D\lambda} = 0 , \tag{2.19}$$

where λ is an affine parameter along the worldlines of fluid particles. Eq. (2.19) is decomposed into the geodesic equation

$$\frac{Dv^a}{D\lambda} \equiv v^b \nabla_b v^a = \frac{d^2 x^a}{d\lambda^2} + \Gamma^a_{bc} \frac{dx^b}{d\lambda} \frac{dx^c}{d\lambda} = 0 \tag{2.20}$$

and the modified conservation equation

$$\frac{d\rho^{(m)}}{d\lambda} + \rho^{(m)} \nabla^b v_b + \frac{2\kappa\xi\phi\, \rho^{(m)}}{1-\kappa\xi\phi^2} \frac{d\phi}{d\lambda} = 0 . \tag{2.21}$$

Test particles move on geodesics, thus verifying the geodesic hypothesis[7] [4]. The modified conservation equation (2.21) is more transparent in the weak field limit, in which it reduces to

$$\frac{\partial\rho^{(m)}}{\partial t} + \vec{\nabla} \cdot \left(\rho^{(m)} \vec{v} \right) + \frac{2\kappa\xi\phi}{1-\kappa\xi\phi^2} \left(\frac{\partial\phi}{\partial t} + \vec{\nabla}\phi \cdot \vec{v} \right) \rho^{(m)} = 0 , \tag{2.22}$$

with $\vec{v}$ denoting the three-dimensional velocity of the non-relativistic fluid; even in the slow-motion limit, the usual energy conservation law $\partial\rho^{(m)}/\partial t + \vec{\nabla} \cdot \left(\rho^{(m)} \vec{v} \right) = 0$ acquires a correction.

[7] This is not the case, considered later in this paper, of Brans-Dicke theory reformulated in the Einstein conformal frame [27], due to an explicit coupling between matter and the Einstein frame scalar.

In the effective coupling approach, the total energy density seen by an observer with four-velocity u^c is

$$\rho^{(II)}_{(total)} \equiv T^{(II)}_{ab} u^a u^b + T^{(m)}_{ab} u^a u^b = \rho^{(II)}_{\phi} + \rho^{(m)} \,, \tag{2.23}$$

where

$$\rho^{(II)}_{\phi} = (u^c \nabla_c \phi)^2 + \frac{1}{2} \nabla^c \phi \nabla_c \phi + V(\phi) - \xi \Box (\phi)^2 - \xi u^a u^b \nabla_a \nabla_b (\phi^2) = \rho^{(I)}_{\phi} - \xi \phi^2 G_{ab} u^a u^b \tag{2.24}$$

or, using eq. (2.13),

$$\rho^{(II)}_{\phi} = \rho^{(I)}_{\phi} \left(1 - \kappa \xi \phi^2 \right) = \kappa \xi \phi^2 \rho^{(m)} \,. \tag{2.25}$$

In the presence of ordinary matter, $\rho^{(II)}_{\phi}$ is a mixture of $\rho^{(I)}_{\phi}$ and of $\rho^{(m)}$ weighted by the factor $\kappa \xi \phi^2$.

2.3 The mixed approach

The third and last possibility to rewrite the field equations (2.1) is to bring the $\xi \phi^2 G_{ab}$ term to the left hand side but to keep the usual, constant, gravitational coupling, thereby obtaining

$$G_{ab} = \kappa \left(T^{(III)}_{ab} + \frac{T^{(m)}_{ab}}{1 - \kappa \xi \phi^2} \right) \,, \tag{2.26}$$

where

$$T^{(III)}_{ab} = \frac{1}{1 - \kappa \xi \phi^2} \left[\nabla_a \phi \nabla_b \phi - \frac{1}{2} g_{ab} \nabla^c \phi \nabla_c \phi - V g_{ab} + \xi \left(g_{ab} \Box - \nabla_a \nabla_b \right) (\phi^2) \right] = \frac{T^{(II)}_{ab}}{1 - \kappa \xi \phi^2} \,, \tag{2.27}$$

Clearly, the limitations due to division by the factor $\left(1 - \kappa \xi \phi^2\right)$ are present also in this mixed approach. Note that

$$\kappa T^{(III)}_{ab} = \kappa_{eff} T^{(II)}_{ab} \tag{2.28}$$

and that the total energy density measured by an observer with four-velocity u^c is

$$\rho^{(III)}_{(total)} \equiv T^{(II)}_{ab} u^a u^b + T^{(m)}_{ab} u^a u^b = \rho^{(II)}_{\phi} + \frac{\rho^{(m)}}{1 - \kappa \xi \phi^2} \,. \tag{2.29}$$

In the absence of ordinary matter, $\rho^{(III)}_{\phi} = \rho^{(II)}_{\phi}$. This time the contracted Bianchi identities $\nabla^b G_{ab} = 0$ yield conservation of the *total* energy-momentum tensor

$$T^{(total)}_{ab} = T^{(II)}_{ab} + \frac{T^{(m)}_{ab}}{1 - \kappa \xi \phi^2} \,, \tag{2.30}$$

$$\nabla^b T_{ab}^{(total)} = 0 \;, \tag{2.31}$$

but $T_{ab}^{(III)}$ is not covariantly conserved:

$$\nabla^b T_{ab}^{(III)} = \frac{-2\kappa\xi\phi}{\left(1-\kappa\xi\phi^2\right)^2} \left(\nabla^b \phi\right) T_{ab}^{(m)} \; . \tag{2.32}$$

However, in the absence of ordinary matter, $T_{ab}^{(III)}$ is conserved, $\nabla^b T_{ab}^{(III)} = 0$.

2.4 Discussion

The three possible cases are summarized in Table 1, and they trivially coincide for $\xi = 0$. For $\xi < 0$, the scalar field ϕ does not possess the critical values (2.16) and the three approaches discussed above are mathematically equivalent, since they differ only by legitimate algebraic manipulations of the field equations (2.1). However, they are physically inequivalent; in fact the identification of the energy density, pressure, and effective equation of state differs in the three approaches, and this is important in the light of the present debate on energy conditions and the second law of thermodynamics with nonminimally coupled scalar fields as the source of gravity [21, 22, 32, 24]. Moreover, the different conservation laws (2.7), (2.18) and (2.32) lead to different physical interpretations. For example, one encounters models of quintessence with NMC [3] and observational constraints are often stated in terms of the effective equation of state. However, when the definitions of energy density and pressure are ambiguous, also the concept of equation of state becomes fuzzy.

In order to compare the different scalar field energy densities we assume, in the rest of this paper, that ϕ is the only form of matter. Besides the obvious simplification in eqs. (2.25) and (2.29), this situation is appropriate for the description of inflationary scenarios of the early universe [2], for late quintessence-dominated cosmological scenarios [3], and for wormhole models [33, 20, 22].

For $\xi < 0$, the effective coupling κ_{eff} is always positive, and $\rho_\phi^{(II)}$ has the same sign as $\rho_\phi^{(III)} = \rho_\phi^{(I)}$.

For $\xi > 0$, in addition to the physical inequivalence, the three approaches are also mathematically inequivalent, as discussed above, and the scalar field possesses the critical values (2.16). If $|\phi| < 1/\sqrt{\kappa\xi}$, then $\kappa_{eff} > 0$ and $\rho_\phi^{(II)}$ has the same sign of $\rho_\phi^{(I)} = \rho_\phi^{(III)}$. If instead $|\phi| > 1/\sqrt{\kappa\xi}$, the effective coupling κ_{eff} is negative, a regime that has been called "antigravity" and studied in the literature [34]. In this regime $\text{sign}(\rho_\phi^{(II)}) = -\text{sign}(\rho_\phi^{(I)}) = -\text{sign}(\rho_\phi^{(III)})$. However, $\kappa_{eff}\,\rho_\phi^{(II)} = \kappa\,\rho_\phi^{(I)} = \kappa\,\rho_\phi^{(III)}$; while it is this product that ultimately enters the field equations, the worries about the negative energy fluxes associated to NMC and leading to time machines and violations of the second law of thermodynamics [24] are expressed using $T_{ab}^{(III)}$

of our notations. Had one proceeded using $T_{ab}^{(II)}$ instead, one would have found, for $\xi > 0$ and $|\phi| > |\phi_c|$, *positive* energy density, but the same wormhole solutions.

The minimal conclusion that one draws from this discussion is that the definition of energy density and flux must be carefully specified when discussing energy conditions in the presence of NMC; and the situation in this respect is now clarified. Another relevant issue is whether there is a physically preferred stress-energy tensor in the NMC theory described by (1.1). The answer is, to a certain extent, a matter of taste, and depends on the task pursued; from a general point of view, the approach à la Callan-Coleman-Jackiw presents certain advantages. First, the corresponding stress-energy tensor $T_{ab}^{(I)}$ is always covariantly conserved, even in the presence of ordinary matter $T_{ab}^{(m)}$; second, this approach does not lead to loss of generality of the solutions. Third, there are situations in cosmology (described in the next section) in which the energy density $\rho_\phi^{(I)}$ is automatically positive-definite, while $\rho_\phi^{(II)}$ is not. However, we cannot provide definitive arguments to rule out the effective and the mixed coupling approaches; we believe that such an argument cannot be presented, and the final decision on what approach is most convenient is left to the reader.

3 Energy conditions in FLRW cosmology

That the energy density of a nonminimally coupled scalar field can be negative has been known for a long time (e.g. [35, 36, 21]), although the troublesome consequences have only recently been emphasized [22, 24]. There are however situations in which negative energies do not occur with the definition of energy density $\rho_\phi^{(I)}$ proposed as the physical one in the previous section.

Let us consider a Friedmann-Lemaitre-Robertson-Walker cosmology described by the line element

$$ds^2 = -dt^2 + a^2(t) \left[\frac{dr^2}{1 - Kr^2} + r^2 \left(d\theta^2 + \sin^2\theta \, d\varphi^2 \right) \right] \tag{3.1}$$

in comoving coordinates (t, r, θ, φ); it is assumed that the dynamics of the universe are driven by the nonminimally coupled scalar field (as is the case, for example, during inflation or in a late quintessence-dominated era). The scale factor $a(t)$ and the scalar field $\phi(t)$ satisfy the Einstein-Friedmann equations

$$\frac{\ddot{a}}{a} = \dot{H} + H^2 = -\frac{\kappa}{6} \left(\rho^{(I)} + 3P^{(I)} \right) \tag{3.2}$$

$$H^2 = \frac{\kappa}{3} \rho^{(I)} - \frac{K}{3a^2} \tag{3.3}$$

(where $P^{(I)}$ is the isotropic pressure obtained from $T_{ab}^{(I)}$ for a time dependent scalar $\phi(t)$). The Hamiltonian constraint (3.3) implies that the energy density $\rho_\phi^{(I)}$ is automatically non-negative for any solutions $(a(t), \phi(t))$ of the Einstein equations for spatially flat ($K = 0$) and for closed ($K = +1$) universes. This conclusion holds in spite of the complicacy of the expressions for $\rho^{(I)}(t)$ and $P^{(I)}(t)$,

$$\rho^{(I)}(t) = \frac{1}{2}\dot{\phi}^2 + 3\xi H^2\phi^2 + 6\xi H\phi\dot{\phi} + V(\phi) \ , \tag{3.4}$$

$$P^{(I)}(t) = \left(\frac{1}{2} - 2\xi\right)\dot{\phi}^2 + 2\xi H\phi\dot{\phi} + 2\xi(6\xi - 1)\dot{H}\phi^2 + 3\xi(8\xi - 1)H^2\phi^2 + 2\xi\phi\frac{dV}{d\phi} \ , \tag{3.5}$$

from which nothing could *a priori* be concluded.

4 Negative energies

The problem of negative energy fluxes with nonminimally coupled scalars already appears with a flat background spacetime, as demonstrated by the simple example of Ref. [24]. The negative energy flux problem is by no means restricted to NMC [27]; here we present an example, analogous to that of Ref. [24], in which weak gravitational waves on a flat background in Brans-Dicke theory exhibit negative energies on a macroscopic time scale. Scalar gravitational waves are currently under investigation by many authors [37].[8]

In the usual formulation of Brans-Dicke theory in the so-called Jordan conformal frame the action is

$$S_{BD} = \int d^4x \sqrt{-g}\left[\phi_{BD}R - \frac{\omega}{2}\, g^{ab}\,\partial_a\phi_{BD}\,\partial_b\phi_{BD}\right] + S_{(m)} \ , \tag{4.1}$$

which exhibits an explicit coupling betweeen the Brans-Dicke field ϕ_{BD} and the Ricci curvature.

Let us consider, in the weak field limit, scalar-tensor gravitational waves around a flat background, described by

$$g_{ab} = \eta_{ab} + h_{ab} \ , \qquad\qquad \phi_{BD} = \phi_0 + \varphi \ , \tag{4.2}$$

where O(h_{ab}) =O(φ/ϕ_0) =O(ϵ), where ϵ is a small parameter. By linearizing the Brans-Dicke field equations in a region outside the sources, one obtains (e.g. [10])

$$R_{ab} = \frac{\partial_a\partial_b\varphi}{\phi_0} + \mathrm{O}(\epsilon^2) \equiv T_{ab}^{(J)}[\varphi] + \mathrm{O}(\epsilon^2) \ , \tag{4.3}$$

$$\Box\varphi = 0 \ . \tag{4.4}$$

The energy density of gravitational waves is given, to order ϵ, only by the scalar waves. The tensor modes give a contribution described by the Isaacson effective stress-energy tensor [40],

[8] For a recent review of the status of gravitational wave detectors, see [38]; see also [39].

which is only of second order. The energy density measurable by an observer with four-velocity u^a is

$$\rho^{(J)} = T_{ab}^{(J)} [\varphi] \, u^a u^b + \mathrm{O}(\epsilon^2) \tag{4.5}$$

and its sign is indefinite. By taking, for example, a monochromatic scalar wave $\varphi = \varphi_0 \cos(k_c x^c)$, one has the oscillating density $\rho^{(J)} = -(k_c x^c)^2 \, \varphi/\phi_0$. Hence, the energy of gravitational waves emitted by a monochromatic source like a binary system is negative over time intervals equal to half of the orbital period, which typically is on the scale of days or months. It is hard to accept that a binary stellar system gains energy by *emitting* gravitational waves.

The root of the problem lies in the non-canonical form of the scalar field energy density in Brans-Dicke theory (in its Jordan frame formulation used so far): T_{ab} has a part that is proportional to the second covariant derivative $\nabla_a \nabla_b \phi_{BD}$, instead of being quadratic in the first derivatives, $\nabla_a \phi_{BD} \nabla_b \phi_{BD}$. $\nabla_a \nabla_b \phi_{BD}$ is the only first order term that survives in the weak field limit, giving the energy density (4.5). A comparison with eq. (2.4) shows that this kind of terms also appears in the energy-momentum tensors $T_{ab}^{(I)}$, $T_{ab}^{(II)}$ and $T_{ab}^{(III)}$ for a nonminimally coupled scalar. It is not surprising that there is a common root for the energy problem in NMC and in Brans-Dicke theory, since the action (1.1) can be rewritten as that for a scalar-tensor theory (4.1), with a ϕ-dependent Brans-Dicke parameter $\omega(\phi)$.

The usual way to cure the problem of negative energy fluxes is to reformulate the theory in the Einstein conformal frame (see Ref. [27] for a review); we illustrate the procedure for the previous example of weak field Brans-Dicke gravitational waves.

One performs the conformal transformation of the metric

$$g_{ab} \longrightarrow \tilde{g}_{ab} = \Omega^2 g_{ab} \, , \qquad \Omega = \sqrt{G\phi} \tag{4.6}$$

and redefines the Brans-Dicke scalar according to

$$\phi_{BD} \longrightarrow \tilde{\phi} = \left(\frac{2\omega + 3}{16\pi G} \right)^{1/2} \ln \left(\frac{\phi_{BD}}{\phi_0} \right) \, . \tag{4.7}$$

The Einstein conformal frame is the set of variables $\left(\tilde{g}_{ab}, \tilde{\phi} \right)$, as opposed to the Jordan conformal frame (g_{ab}, ϕ_{BD}). In the Einstein frame, scalar-tensor gravitational waves are described by

$$\tilde{g}_{ab} = \eta_{ab} + \tilde{h}_{ab} \, , \qquad \tilde{h}_{ab} = h_{ab} + \frac{\varphi}{\phi_0} \eta_{ab} \, , \qquad \tilde{\phi} = \tilde{\phi}_0 + \tilde{\varphi} \tag{4.8}$$

where, according to eq. (4.7),

$$\tilde{\varphi} = \left(\frac{2\omega + 3}{16\pi G} \right)^{1/2} \varphi \, . \tag{4.9}$$

In regions of spacetime outside sources, the Einstein frame linearized field equations are

$$\tilde{R}_{ab} - \frac{1}{2} \tilde{g}_{ab} \tilde{R} = \kappa \left\{ \tilde{T}_{ab} [\tilde{\varphi}] + T_{ab}^{(eff)} \left[\tilde{h}_{cd} \right] \right\} \, , \tag{4.10}$$

$$\Box\tilde{\varphi} = 0 \; , \tag{4.11}$$

where

$$\tilde{T}_{ab}\left[\tilde{\varphi}\right] = \partial_a\tilde{\varphi}\,\partial_b\tilde{\varphi} - \frac{1}{2}\,\eta_{ab}\,\partial^c\tilde{\varphi}\,\partial_c\tilde{\varphi} \tag{4.12}$$

and $T_{ab}^{(eff)}\left[\tilde{h}_{cd}\right]$ is Isaacsons's effective stress-energy tensor [40] associated to the tensor modes $\tilde{h}_{cd}$. In the Einstein frame description, the energy momentum tensor of the scalar field $\tilde{T}_{ab}\left[\tilde{\varphi}\right]$ has the canonical quadratic dependence on $\partial_d\tilde{\varphi}$, and both scalar and tensor modes give contributions of the same order (that is, of second order) to the field equations. For a monochromatic plane wave $\tilde{\varphi} = \tilde{\varphi}_0 \cos\left(l^c x_c\right)$, one has now

$$\tilde{\rho} = \tilde{T}_{ab}u^a u^b = \left(l_a u^a \tilde{\varphi}\right)^2 + T_{ab}^{(eff)} u^a u^b \geq 0 \; , \tag{4.13}$$

with a positive-definite contribution from the scalar modes.

5 NMC and gravitational waves

Due to the explicit coupling between the gravitational and the scalar fields in the NMC theory (1.1), when gravitational waves are excited, also scalar waves are generated. We set $V(\phi) = 0$ and $T_{ab}^{(m)} = 0$. The stress-energy tensor (2.4) of ϕ contains the term $\xi\left(g_{ab}\Box - \nabla_a\nabla_b\right)\left(\phi^2\right)$ linear in the second derivatives of ϕ instead of a canonical term quadratic in its first derivatives $\nabla_c\phi$: this non-canonical structure is analogous to that of the effective stress-energy tensor of the Brans-Dicke scalar. Therefore, in principle, one could encounter here the same problems discussed for scalar-tensor gravitational waves in Brans-Dicke theory. However, this is not the case, at least in the first order perturbation analysis considered in this paper, as is shown below.

One expands the metric and the scalar field around their flat space values,

$$g_{ab} = \eta_{ab} + h_{ab} \; , \qquad\qquad \phi = \phi_0 + \psi \; , \tag{5.1}$$

with ϕ_0 =const. (solutions with constant scalar field are discussed in the Appendix), and $\mathrm{O}(h_{ab})$ $=\mathrm{O}(\psi)$ $=\mathrm{O}(\epsilon)$. It is easily seen from the field equations (2.1) that the requirement that spacetime be flat implies ϕ =constant. The offending term in $T_{ab}^{(I)}$ then reduces to $\xi\left(g_{ab}\Box - \nabla_a\nabla_b\right)\left(\phi^2\right) = -2\xi\phi_0\,\partial_a\partial_b\psi + \mathrm{O}(\epsilon^2)$, where we used the equation

$$\Box\psi = 0 + \mathrm{O}(\epsilon^2) \; , \tag{5.2}$$

which follows from the trace of the field equations

$$R = -6\xi\phi_0\kappa\,\Box\psi + \mathrm{O}(\epsilon^2) \; , \tag{5.3}$$

and from the Klein-Gordon equation

$$\Box\phi - \xi R\phi = 0 \; . \tag{5.4}$$

Hence, the negative energy problems seem to be present; however, in NMC theory, contrarily to Brans-Dicke theory, the only value of the constant ϕ compatible with a flat background is $\phi_0 = 0$. In fact, a constant non-vanishing scalar field corresponds to Schwarzschild and anti-Schwarzschild solutions [22]. By setting $\phi_0 = 0$ as required for a flat background, the troubles disappear since $T_{ab}^{(I)}$ reduces, to first order, to a canonical form quadratic in the first derivatives of the field, and the energy density of scalar modes in NMC theory is positive definite.

6 Discussion and conclusions

We discussed the possible mathematical definitions of energy density and pressure of a non-minimally coupled scalar field, and the ambiguities arising when one studies the weak and null energy conditions for such a field. Three inequivalent definitions of ρ and P are possible.

¿From a fundamental point of view, the difficulty in identifying the "physically correct" stress-energy tensor hints at the problem of the localization of gravitational energy. In fact, one can regard the physical system described by the action (1.1) as two coupled subsystems, the gravitational tensor field g_{ab} and the (non-gravitational) scalar field ϕ, with the term $-\sqrt{-g}\xi\phi^2 R/2$ in the Lagrangian density as an explicit interaction term.

While no general prescription is known for the energy density of the gravitational field [4, 40], a totally satisfactory definition of stress-energy tensor is available for a minimally coupled ($\xi = 0$) scalar field. One should recognize that, when $\xi = 0$, g_{ab} and ϕ exchange energy and momentum during their dynamical evolution; as a consequence, the stress-energy tensor of ϕ contains terms describing this interaction. The impossibility of localizing the gravitational field energy may therefore be the source of the problems with NMC. Then, the reported violations of the second law of thermodynamics by NMC [24] are not surprising since the scalar ϕ under study is not an isolated system, but rather a subsystem coupled to a second one, and we know nothing about the energy density and entropy of the latter. The second law refers to the isolated global system, not to a subsystem of it.

A similar problem affects other theories of gravity in which a scalar field explicitly couples to gravity: string theories, supergravity, Brans-Dicke theory, scalar-tensor and higher derivative theories; negative kinetic energy densities appear. This is problematic when the kinetic terms dominate the dynamics, as in the example of Sec. 4. In scalar-tensor theories of gravity the scalar field itself is a gravitational field, and its stress-energy tensor contains a part of the gravitational energy density.

The usual cure for the problem has been the reformulation of the theory in the Einstein conformal frame [1, 24, 27]. This way out of the problem leaves many authors dissatisfied because it involves a radical change of the theory, thereby losing much of its original motivation. We feel that a completely satisfactory answer to the problem cannot come without a successful solution of the gravitational energy localization problem in general relativity.

It is at present unclear whether the view of NMC as the physics of two coupled subsystems may be useful in the quest for an approximate localization of gravitational energy.

Appendix: solutions with constant scalar field

Let us adopt approach I and assume that $T_{ab}^{(m)} = 0$; then the field equations are

$$G_{ab} = \kappa T_{ab}^{(I)} = \kappa \left[\partial_a \phi \, \partial_b \phi - \frac{1}{2} g_{ab} \, \partial^c \phi \, \partial_c \phi - V(\phi) \, g_{ab} + \xi \left(g_{ab} \Box - \nabla_a \nabla_b \right) (\phi^2) - \xi \phi^2 G_{ab} \right] , \tag{A.1}$$

$$\Box \phi - \xi R \phi - \frac{dV}{d\phi} = 0 \; . \tag{A.2}$$

By assuming that $\phi = \text{const.} \equiv \phi_0$, one has

$$G_{ab} = -\kappa V_0 g_{ab} + \kappa \xi \phi_0^2 G_{ab} \; , \tag{A.3}$$

$$\xi R \phi_0 + V_0' = 0 \; . \tag{A.4}$$

In the non-trivial case $\phi_0 \neq 0$, one has

$$R = -\frac{V_0'}{\xi \phi_0} = \text{const.} \; , \tag{A.5}$$

i.e. all these solutions have constant Ricci curvature. Eq. (A.3) then implies that

$$R = 4\kappa V_0 + \kappa \xi \phi_0^2 R \tag{A.6}$$

and, if $\phi_0^2 \neq 1/\kappa\xi$,

$$R = \frac{4\kappa V_0}{1 - \kappa \xi \phi_0^2} \; . \tag{A.7}$$

By comparing eqs. (A.5) and (A.7), one obtains the constraint between V, ϕ_0, and ξ

$$\phi_0 V_0 = \frac{-V_0'}{4\kappa\xi \left(1 - \kappa \xi \phi_0^2 \right)} \; . \tag{A.8}$$

If instead $\phi^2 = \phi_c^2 \equiv 1/(\kappa\xi)$ with $\xi > 0$, then it must be $V_0 = 0$, $R = 0$, and $V_0' = 0$; the potential $V(\phi)$ must have a zero and horizontal tangent in $\pm\phi_c$. These critical scalar field values have been studied in scalar field cosmology for the potential

$$V(\phi) = \frac{\alpha}{2} \phi^2 \left(\frac{6}{\kappa} - \phi^2 \right) \tag{A.9}$$

and for $\xi = 1/6$; in this case all the solutions with constant Ricci curvature are classified [41]. For Lorentzian wormholes without potential V, all the solutions corresponding to the critical scalar field values are also classified [22].

References

[1] C.H. Brans and R.H. Dicke, *Phys. Rev.* **124**, 925 (1961).

[2] A.R. Liddle and D.H. Lyth, *Cosmological Inflation and Large Scale Structure* (Cambridge University Press, Cambridge, 2000); E.W. Kolb and M.S. Turner, *The Early Universe* (Addison-Wesley, Mass., 1994); A.D. Linde, *Particle Physics and Inflationary Cosmology* (Hardwood Academic, Chur, Switzerland, 1990).

[3] J.-P. Uzan, *Phys. Rev. D* **59**, 123510 (1999); F. Perrotta, C. Baccigalupi and S. Matarrese, *Phys. Rev. D* **61**, 023507 (1999); T. Chiba, *Phys. Rev. D* **60**, 083508 (1999); C. Baccigalupi, F. Perrotta and S. Matarrese, preprint astro-ph/0005543.

[4] R.M. Wald, *General Relativity* (The University of Chicago Press, Chicago, 1984).

[5] L.H. Ford and D.J. Toms, *Phys. Rev. D* **25**, 1510 (1982).

[6] I.L. Buchbinder and S.D. Odintsov, *Sov. J. Nucl. Phys.* **40**, 848 (1983); I.L. Buchbinder and S.D. Odintsov, *Lett. Nuovo Cimento* **42**, 379 (1985); I.L. Buchbinder, *Fortschr. Phys.* **34**, 605 (1986); I.L. Buchbinder, S.D. Odintsov and I.L. Shapiro, in *Group-Theoretical Methods in Physics*, M. Markov ed. (Nauka, Moscow, 1986); S.D. Odintsov, *Fortschr. Phys.* **39**, 621 (1991); T.S. Muta and S.D. Odintsov, *Mod. Phys. Lett. A* **6**, 3641 (1991); E. Elizalde and S.D. Odintsov, *Phys. Lett. B* **333**, 331 (1994).

[7] B. Allen, *Nucl. Phys. B* **226**, 232 (1983); J. Ishikawa, *Phys. Rev. D* **28**, 2445 (1983).

[8] V. Faraoni, preprint hep-th/0009053 (2000).

[9] V. Faraoni, *Phys. Rev. D* **53**, 6813 (1996).

[10] C.M. Will, *Theory and Experiment in Gravitational Physics*, revised edition (Cambridge University Press, Cambridge, 1993).

[11] S. Bellucci and A. Shiekh, gr-qc/9701065; ibid. gr-qc/9703085.

[12] S. Bellucci and V. Faraoni, *Phys. Rev. D* **49**, 2922 (1994); *Phys. Lett. B* **377**, 55 (1996); S. Bellucci, *Proceedings of the International Europhysics Conference on High Energy Physics*, (Jerusalem, 19-26 August 1997), edited by D. Lellouch, G. Mikenberg, E. Rabinovici, (Springer, Berlin), page 918, hep/ph-9710562.

[13] S. Sonego and V. Faraoni, *Class. Quant. Grav.* **10**, 1185 (1993).

[14] A.A. Grib and E.A. Poberii, *Helv. Phys. Acta* **68**, 380 (1995); A.A. Grib and W.A. Rodrigues, *Gravit. Cosmol.* **1**, 273 (1995).

[15] S. Bellucci, *Phys. Rev. D* **57**, 1057 (1998); *Fortschr. Phys.* **40**, 393 (1992); *Prog. Teor. Phys.* **78**, 1176 (1987); see also S. Bellucci and D. O'Reilly, *Nucl. Phys. B* **364**, 495 (1991).

[16] T. Futamase and K. Maeda, *Phys. Rev. D* **39**, 399 (1989).

[17] A.A. Starobinsky, *Sov. Astron. Lett.* **7**, 36 (1981); M. Madsen, *Class. Quant. Grav.* **5**, 627 (1988); T. Futamase, T. Rothman and R. Matzner, *Phys. Rev. D* **39**, 405 (1989); N. Makino and M. Sasaki, *Prog. Theor. Phys.* **86**, 103 (1991); A. Barroso *et al.*, *Phys. Lett. B* **275**, 264 (1992); R. Fakir and S. Habib, *Mod. Phys. Lett. A* **8**, 2827 (1993); A.M. Laycock and A.R. Liddle, *Phys. Rev. D* **49**, 1827 (1994); J. Garcia-Bellido and A. Linde, *Phys. Rev. D* **52**, 6370 (1995); E. Komatsu and T. Futamase, *Phys. Rev. D* **58**, 023004 (1998); B. Bassett and S. Liberati, *Phys. Rev. D* **58**, 021302 (1998); T. Futamase and M. Tanaka, *Phys. Rev. D* **60**, 063511 (1999); E. Komatsu and T. Futamase, *Phys. Rev. D* **59**, 064029 (1999); J. Lee *et al.*, *Phys. Rev. D* **61**, 027301 (1999).

[18] A.O. Barvinsky, A. Yu. Kamenshchik and I.V. Mishakov, *Nucl. Phys. B* **491**, 378 (1977); R. Fakir, *Phys. Rev. D* **41**, 3012 (1990); A. Yu. Kamenshchik, I.M. Khalatnikov and A.V. Toporensky, *Phys. Lett. B* **357**, 35 (1995); A.O. Barvinsky, *Nucl. Phys. B* **561**, 159 (1999); C. da Silva and R.M. Williams, *Class. Quant. Grav.* **17**, 1827 (2000).

[19] P. Jetzer, *Phys. Rep.* **220**, 1636 (1992); J.J. van der Bij and M. Gleiser, *Phys. Lett. B* **194**, 482 (1987); A.R. Liddle and M.S. Madsen, *Int. J. Mod. Phys. D* **1**, 101 (1992).

[20] M. Visser, *Lorentzian Wormholes: from Einstein to Hawking* (American Institute of Physics Press, Woodbury, New York, 1995).

[21] W.A. Hiscock, *Class. Quant. Grav.* **7**, L35 (1990).

[22] C. Barcelo and M. Visser, *Class. Quant. Grav.* **17**, 3843 (2000).

[23] H. Bondi, *Rev. Mod. Phys.* **29**, 423 (1957); M. Alcubierre, *Class. Quant. Grav.* **11**, L73 (1994).

[24] L.H. Ford and T.A. Roman, preprint gr-qc/0009076.

[25] J. Garriga and T. Tanaka, *Phys. Rev. Lett.* **84**, 2778 (2000).

[26] C. Barcelo and M. Visser, *Nucl. Phys. B* **584**, 415 (2000).

[27] V. Faraoni, E. Gunzig and P. Nardone, *Fundam. Cosm. Phys.* **20**, 121 (1999).

[28] A. Akhundov, S. Bellucci and A. Shiekh, *Phys. Lett. B* **395**, 16 (1997).

[29] C.G. Callan Jr., S. Coleman and R. Jackiw, *Ann. Phys. (NY)* **59**, 42 (1970).

[30] N.A. Chernikov and E.A. Tagirov, *Ann. Inst. H. Poincaré A* **9**, 109 (1968).

[31] E. Gunzig *et al.*, *Phys. Rev. D* **63**, 067301 (2001); preprint gr-qc/0012085; V. Faraoni, *Phys. Rev. D* **62**, 023504 (2000).

[32] M. Visser and C. Barcelo, preprint gr-qc/0001099 (2000).

[33] J.J. Halliwell and R. Laflamme, *Class. Quant. Grav.* **6**, 1839 (1989); D.H. Coule and K. Maeda, *Class. Quant. Grav.* **7**, 955 (1990); D.H. Coule, *Class. Quant. Grav.* **9**, 2353 (1992).

[34] A.A. Starobinsky, *Sov. Astron. Lett.* **7**, 36 (1981); M.D. Pollock, *Phys. Lett. B* **108**, 386 (1982); E. Gunzig and P. Nardone, *Phys. Lett. B* **134**, 412 (1984).

[35] J.D. Bekenstein, *Phys. Rev. D* **11**, 2072 (1975); L.H. Ford, *Phys. Rev. D* **35**, 2339 (1987).

[36] M. Madsen, *Class. Quant. Grav.* **5**, 627 (1988).

[37] S. Bellucci, V. Faraoni and D. Babusci, *Phys. Lett. A* **282**, 357 (2001); M. Maggiore and A. Nicolis, *Phys. Rev. D* **62**, 024004 (2000); M. Gasperini, *Phys. Lett. B* **470**, 67 (1999); M. Shibata, K. Nakao and T. Nakamura, *Phys. Rev. D* **50**, 7304 (1994).

[38] *Proceedings of the Third Edoardo Amaldi Conference*, S. Meshkov *et al.* eds. (Pasadena, 1999), in press.

[39] B. Caron *et al.*, *1st International LISA Symposium on Gravitational Waves*, (Oxfordshire, England, 9-12 Jul 1996), *Class. Quant. Grav.* **14** 1461 (1997).

[40] C.W. Misner, K.S. Thorne and J.A. Wheeler, *Gravitation* (Cambridge University Press, Cambridge, 1973).

[41] E. Gunzig *et al.*, *Class. Quant. Grav.* **17**, 1783 (2000).

	(I) approach á la CCJ	(II) effective coupling approach	(III) mixed approach
field equations	$G_{ab}=\kappa\left(T_{ab}^{(I)}+T_{ab}^{(m)}\right)$	$G_{ab}=\kappa_{eff}\left(T_{ab}^{(II)}+T_{ab}^{(m)}\right)$	$G_{ab}=\kappa\left(T_{ab}^{(III)}+\frac{T_{ab}^{(m)}}{1-\kappa\xi\phi^2}\right)$
T_{ab}	$T_{ab}^{(I)}=\partial_a\phi\partial_b\phi-\frac{1}{2}g_{ab}\partial^c\phi\partial_c\phi$ $+\xi\left(g_{ab}\Box-\nabla_a\nabla_b\right)(\phi^2)$ $-Vg_{ab}+\xi\phi^2G_{ab}$	$T_{ab}^{(II)}=T_{ab}^{(I)}-\xi\phi^2G_{ab}$	$T_{ab}^{(III)}=\frac{T_{ab}^{(II)}}{1-\kappa\xi\phi^2}$
conservation law	$\nabla^bT_{ab}^{(I)}=0$	$\nabla^bT_{ab}^{(II)}=\frac{-2\kappa\xi\phi}{1-\kappa\xi\phi^2}\cdot$ $\left(T_{ab}^{(II)}+T_{ab}^{(m)}\right)\nabla^b\phi$	$\nabla^bT_{ab}^{(III)}=\frac{-2\kappa\xi\phi}{1-\kappa\xi\phi^2}T_{ab}^{(m)}\nabla^b\phi$
energy density	$\rho_\phi^{(I)}=(u^c\partial_c\phi)^2+\frac{1}{2}\partial^c\phi\partial_c\phi$ $-\xi u^au^b\nabla_a\nabla_b(\phi^2)$ $+V-\xi\Box(\phi^2)+\xi\phi^2G_{ab}u^au^b$	$\rho_\phi^{(II)}=\rho_\phi^{(I)}\left(1-\kappa\xi\phi^2\right)$	$\rho_\phi^{(III)}=\rho_\phi^{(II)}$

Table 1: a comparison of the three possible approaches to NMC.